# NONLINEAR ABSORPTION POWER AND LINEWIDTHS IN QUANTUM WELL WITH TRIANGULAR POTENTIAL

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**Abstract:** The analytic expression for absorption power of a strong electromagnetic waves caused by confined electrons in quantum well is obtained in the case of electron-optical phonon scattering. Nonlinear optically detected electrophonon resonance (ODEPR) effect in a specific GaAs/AlAs quantum well with triangular potential is investigated. Conditions for ODEPR are discussed based on the curves expressing the dependence of absorption power on the photon energy. From these curves we obtained ODEPR- linewidths as profiles of the curves. Computational results show that the nonlinear ODEPR- linewidths increase with temperature and decrease with electric field amplitude.
**Keywords:** absorption power, quantum well, triangular potential, ODEPR-linewidths

## 1 INTRODUCTION

Optically detected electrophonon resonance linewidths are the good tools for investigating scattering mechanisms of carriers and hence can be used to probe electron-phonon scattering mechanisms. Most of the works on linewidths have focused on the transport properties of low dimensional semiconductors. Unuma *et al.* [1] investigated the intersubband absorption linewidths in GaAs quantum well for various kinds of scattering mechanisms. The obtained results of this work showed that the linewidths decrease with well width. Kang and co-works [2, 3, 4, 5] used the operator projection technique to study intraband linewidths of the optical conductivity in quantum wells due to LO-phonon scattering. Their works indicated that the linewidths increase with the temperature and decrease with the well widths. Li and Ning [6] investigated the influence of electron-electron and electron-phonon scattering on the linewidths in quantum wells. Their results were in line with experimental data obtained by Unuma *et al.* [7], which showed the increase of linewidths with temperature. Lee *et al.* applied the continued-fraction-based theory to investigate the dependence of the linewidths on the temperature in GaAs and CdS [8]. Their results showed that, the linewidths increase with the temperature. The study of linewidths, however, has mostly focused on linear ODEPR -

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linewidth, still limited on nonlinear ones. Therefore, nonlinear ODEPR- linewidth effect in quantum wells is needed for further study.

Recently, our group has proposed a computational method to obtain linewidths from graphs of $P(\omega)$ [9], and used this method to determine cyclotron resonance linewidths in CQW [10], and in GaAs/AlAs quantum wires [11]. In this paper, we investigate the nonlinear absorption power and ODEPR linewidths in the quantum well with triangular potential. First, we derive the analytical expression of nonlinear absorption power. From curves on the graphs of absorption power as a function of photon energy, we obtain ODEPR- linewidths as a profiles of curves by using the profile method presented in one of our previous papers [9]. The dependences of nonlinear linewidth on temperature and the electric field amplitude are discussed.

## 2 MODEL OF A QUANTUM WELL WITH TRIANGULAR POTENTIAL

The electron wave function in quantum well with triangular potential takes the form [12]

$$\begin{aligned} \psi_{\vec{k}_\perp,n}(\vec{r}_\perp,z) &= \frac{1}{\sqrt{L_xL_y}}e^{i\vec{k}_\perp\vec{r}_\perp}\left[\frac{\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}}{Ai'^2(s_0)-s_0Ai^2(s_0)}\right]^{1/2} \\ &\times Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_n}{eE_0}\right)\right], \end{aligned} \tag{1}$$

where $L_x$, $L_y$ are the well's widths in $x$, $y$ dimensions; $\vec{k}_\perp = \vec{k}_x + \vec{k}_y$; $\vec{r}_\perp = x\vec{i} + y\vec{j}$; $m^*$ is the electron effective mass; $e$ is the charge of electron; $E_0$ is electric field amplitude; $s_0 = \left(\frac{2m^*\alpha}{\hbar^2}\right)^{1/3}\left(-\frac{E_n}{\alpha}\right)$; $n$ the radial quantum number ($n = 1, 2, 3, \ldots$); $E_n$ the energy levels in $z$ dimensions which has the form

$$E_n = \left(\frac{\hbar^2}{2m^*}\right)^{1/3}\left[\frac{3\pi eE_0}{2}\left(n-\frac{1}{4}\right)\right]^{2/3} \tag{2}$$

and the Airy function can be given by

$$Ai(s) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3}+st\right)dt. \tag{3}$$

The energy levels of the system can be worked out by

$$E(k_\perp,n) = \frac{\hbar^2k_\perp^2}{2m^*} + \left(\frac{\hbar^2}{2m^*}\right)^{1/3}\left[\frac{3\pi eE_0}{2}\left(n-\frac{1}{4}\right)\right]^{2/3}. \tag{4}$$

The electron form factor is given by [12]

$$I_{n',n} = \left[\frac{\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}}{Ai'^2(s_0)-s_0Ai^2(s_0)}\right]^2\left(\int_0^{L_z} e^{iq_zz}Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_{n'}}{eE_0}\right)\right]\right.$$

$$\times Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_n}{eE_0}\right)\right]dz\Bigg)^2. \quad (5)$$

The integral in Eq.(5) cannot be calculated analytically due to its complexity. However, the physical meaning can be worked out by numerical computation.

## 3 ANALYTICAL RESULTS

When an electromagnetic wave characterized by a time-dependent electric field of amplitude $E_0$ and angular frequency $\omega$ is applied, the nonlinear absorption power delivered to the system is given by [15, 16]

$$\begin{aligned} P_{NLn}(\omega) &= \frac{E_{0z}^2}{2}\mathrm{Re}[\sigma_{NLn}(\omega)] = \frac{E_{0z}^2}{2}\left\{\mathrm{Re}[\sigma_{zz}(\omega)] + \mathrm{Re}[\sigma_{zzz}(\omega)E_{0z}(\omega)]\right\}, \\ &= P_0(\omega) + P_1(\omega). \end{aligned} \quad (6)$$

Here $\sigma_{zz}(\omega)$ is the $z$ component of the optical conductivity tensor. Utilizing the general expression for the conductivity, presented by Kang *el al.* [17] and Lee *et al.* [18], the linear absorption power is given, at the subband edge ($k_z = 0$), by the following expression

$$\begin{aligned} P_0(\omega) &= \frac{E_{0z}^2}{2}\mathrm{Re}[\sigma_{zz}(\omega)] = \frac{e^3E_0^3}{\hbar[Ai'^2(s_0) - s_0Ai^2(s_0)]^2} \\ &\times \sum_{k_\perp,n}\sum_{k'_\perp,n'}\frac{B_0(\omega)}{(\hbar\omega-\Delta E)^2 + B_0^2(\omega)}\delta_{k_\perp,k'_\perp}K_{n,n'}L_{n,n'}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} B_0(\omega) &= \frac{\pi}{f_\beta - f_\alpha}\sum_{\vec{q},\gamma}|C_{\beta\gamma}(\vec{q})|^2\big\{[(1+N_q)f_\gamma(1-f_\alpha) - N_qf_\alpha(1-f_\gamma)]\delta(\hbar\omega - E_{\gamma\alpha} + \hbar\omega_q) \\ &+ [N_qf_\gamma(1-f_\alpha) - (1+N_q)f_\alpha(1-f_\gamma)]\delta(\hbar\omega - E_{\gamma\alpha} - \hbar\omega_q)\big\} \\ &+ \frac{\pi}{f_\beta - f_\alpha}\sum_{\vec{q},\gamma}|C_{\gamma\alpha}(\vec{q})|^2\big\{[(1+N_q)f_\beta(1-f_\gamma) - N_qf_\gamma(1-f_\beta)]\delta(\hbar\omega - E_{\beta\gamma} + \hbar\omega_q) \\ &+ [N_qf_\beta(1-f_\gamma) - (1+N_q)f_\gamma(1-f_\beta)]\delta(\hbar\omega - E_{\beta\gamma} - \hbar\omega_q)\big\}. \end{aligned}$$

In the above equation, we denote $E_{\alpha\beta} = E_\beta - E_\alpha = E_{k'_\perp,n'} - E_{k_\perp,n}$; $E_{k_\perp,n}$ and $E_{k'_\perp,n'}$ are the energy of the electron in the initial and final state, respectively; $f_\alpha(f_\beta)$ is the Fermi-Dirac distribution function of electron with energy $E_\alpha(E_\beta)$; and $K_{n,n'}$, $L_{n,n'}$ are given by

$$K_{n,n'} = \int_{-\infty}^{+\infty} Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_n}{eE_0}\right)\right] \times Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_{n'}}{eE_0}\right)\right]dz; \quad (8)$$

$$L_{n,n'} = \int_{-\infty}^{+\infty} Ai\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_{n'}}{eE_0}\right)\right] Ai'\left[\left(\frac{2m^*eE_0}{\hbar^2}\right)^{1/3}\left(z-\frac{E_n}{eE_0}\right)\right]dz. \quad (9)$$

Transforming the sums over $\vec{q}$ and $\gamma$ in Eq.(8) into integrals, we obtain

$$\begin{aligned}
B_0(\omega) &= \frac{L_x L_y D m^*}{8\pi^2\hbar^2(f_\beta - f_\alpha)} \sum_{n''} \Big\{ \Big( \big[ -\frac{1}{k'_\perp + M_{01}} + \frac{1}{k'_\perp - M_{01}} \big] F_{01} \\
&\quad + \big[ -\frac{1}{k'_\perp + M_{02}} + \frac{1}{k'_\perp - M_{02}} \big] F_{02} \Big) N_1 + \Big( \big[ \frac{1}{-k_\perp + M_{03}} + \frac{1}{k_\perp + M_{03}} \big] F_{03} \\
&\quad + \big[ \frac{1}{-k_\perp + M_{03}} + \frac{1}{k_\perp + M_{03}} \big] F_{04} \Big) N_3 \Big\}; \qquad (10)
\end{aligned}$$

where

$$D = \left[ \frac{e^2 \hbar\omega_{LO}}{2\varepsilon_0 V_0} \left( \frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \right]^{1/2}; \qquad (11)$$

here $\hbar\omega_{LO}$ is the LO-phonon energy, $\varepsilon_0$ is the permittivity of free space, $\chi_\infty$ and $\chi_0$ are the high- and low-frequency dielectric constants, respectively;

$$\begin{aligned}
M_{01,02} &= \left[ k_\perp^2 + \frac{2m^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n''} + E_n) \right]^{1/2}; \\
M_{03,04} &= \left[ k_\perp'^2 - \frac{2m^*}{\hbar^2}(\hbar\omega \pm \hbar\omega_{LO} - E_{n'} + E_{n''}) \right]^{1/2}; \\
F_{01} &= (1 + N_q)(1 - f_\alpha)\big(1 + \exp[\theta(\frac{\hbar^2 M_{01}^2}{2m^*} + E_{n''} - E_F)]\big)^{-1}; \\
F_{02} &= N_q(1 - f_\alpha)\big(1 + \exp[\theta(\frac{\hbar^2 M_{02}^2}{2m^*} + E_{n''} - E_F)]\big)^{-1}; \\
F_{03} &= (1 + N_q) f_\beta \big[1 - \big(1 + \exp[\theta(\frac{\hbar^2 M_{03}^2}{2m^*} + E_{n''} - E_F)]\big)^{-1}\big]; \\
F_{04} &= N_q f_\beta \big[1 - \big(1 + \exp[\theta(\frac{\hbar^2 M_{04}^2}{2m^*} + E_{n''} - E_F)]\big)^{-1}\big]; \\
N_1 &= N_2 = \int_{-\infty}^{+\infty} I_{n',n''} dq_z; N_3 = N_4 = \int_{-\infty}^{+\infty} I_{n,n''} dq_z.
\end{aligned}$$

Inserting Eq.(10) into Eq.(7), we obtain the expression of linear absorption power. The first order nonlinear absorption power is given by the following expression

$$\begin{aligned}
P_1(\omega) &= \frac{E_0^2}{2} \mathrm{Re}[\sigma_{zzz}(\omega) E_0(\omega)] \\
&= \frac{e^3 E_0^3 \hbar}{2m^*} \frac{\left( \frac{2m^* e E_0}{\hbar^2} \right)^{4/3}}{[Ai'^2(s_0) - s_0 Ai^2(s_0)]^3} \sum_{n_\alpha} \sum_{n_\beta} \sum_{n_\gamma} \sum_{n_\delta} \frac{f_{k_{\perp\beta},n_\beta} - f_{k_{\perp\alpha},n_\alpha}}{(\hbar\omega - E_{\beta\alpha})^2 + B_0^2(\omega)} \\
&\times \Big\{ - \frac{[(\hbar\omega - E_{\beta\alpha}) B_1(2\omega) + (2\hbar\omega - E_{\beta\gamma}) B_0(\omega)]}{(2\hbar\omega - E_{\beta\gamma})^2 + [B_1(2\omega)]^2} \\
&+ \frac{[(\hbar\omega - E_{\beta\alpha}) B_2(2\omega) + (2\hbar\omega - E_{\delta\alpha}) B_0(\omega)]}{(2\hbar\omega - E_{\delta\alpha})^2 + [B_2(2\omega)]^2} \Big\}
\end{aligned}$$

$$\begin{aligned}&\times\quad \delta_{k_{\perp\alpha}k_{\perp\beta}}\delta_{k_{\perp\gamma}k_{\perp\alpha}}\delta_{k_{\perp\beta}k_{\perp\gamma}}\delta_{k_{\perp\beta}k_{\perp\delta}}\delta_{k_{\perp\delta}k_{\perp\alpha}}\times K_{n_\alpha n_\beta}K_{n_\gamma n_\alpha}K_{n_\beta n_\gamma}K_{n_\beta n_\delta}K_{n_\delta n_\alpha}\\&\times\quad L_{n_\alpha n_\beta}L_{n_\gamma n_\alpha}L_{n_\beta n_\gamma}L_{n_\beta n_\delta}L_{n_\delta n_\alpha};\end{aligned} \tag{12}$$

where

$$\begin{aligned}B_1(2\omega) &= \frac{L_xL_yDm^*}{8\pi^2\hbar^2(f_\beta-f_\alpha)}\sum_{n_\mu}\Big\{\Big(\big[-\frac{1}{k_{\perp\gamma}+M_{11}}+\frac{1}{k_{\perp\gamma}-M_{11}}\big]F_{11}\\&- \big[-\frac{1}{k_{\perp\gamma}+M_{12}}+\frac{1}{k_{\perp\gamma}-M_{12}}\big]F_{12}-\big[-\frac{1}{k_{\perp\gamma}+M_{13}}+\frac{1}{k_{\perp\gamma}-M_{13}}\big]F_{13}\\&+ \big[-\frac{1}{k_{\perp\gamma}+M_{14}}+\frac{1}{k_{\perp\gamma}-M_{14}}\big]F_{14}\Big)L_1+\Big(\big[\frac{1}{-k_{\perp\beta}+M_{15}}+\frac{1}{k_{\perp\beta}+M_{15}}\big]F_{15}\\&- \big[\frac{1}{-k_{\perp\beta}+M_{16}}+\frac{1}{k_{\perp\beta}+M_{16}}\big]F_{16}\Big)L_2\Big\};\end{aligned} \tag{13}$$

$$\begin{aligned}B_2(2\omega) &= \frac{L_xL_yDm^*}{8\pi^2\hbar^2(f_\beta-f_\alpha)}\sum_{n_\mu}\Big\{\Big(\big[\frac{1}{-k_{\perp\delta}+M_{21}}+\frac{1}{k_{\perp\delta}+M_{21}}\big]F_{21}\\&- \big[\frac{1}{-k_{\perp\delta}+M_{22}}+\frac{1}{k_{\perp\delta}+M_{22}}\big]F_{22}-\big[\frac{1}{-k_{\perp\delta}+M_{23}}+\frac{1}{k_{\perp\delta}+M_{23}}\big]F_{23}\\&+ \big[\frac{1}{-k_{\perp\delta}+M_{24}}+\frac{1}{k_{\perp\delta}+M_{24}}\big]F_{24}\Big)L_3+\Big(-\big[-\frac{1}{k_{\perp\alpha}+M_{25}}+\frac{1}{k_{\perp\alpha}-M_{25}}\big]F_{15}\\&+ \big[-\frac{1}{k_{\perp\alpha}+M_{26}}+\frac{1}{k_{\perp\alpha}-M_{26}}\big]F_{26}\Big)L_4\Big\};\end{aligned} \tag{14}$$

$$\begin{aligned}M_{11,12} &= \left[k_{\perp\beta}^2-\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\beta}+E_{n_\mu})\right]^{1/2},\\
M_{13,14} &= \left[k_{\perp\alpha}^2-\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\beta}+E_{n_\mu})\right]^{1/2},\\
M_{15,16} &= \left[k_{\perp\alpha}^2+\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\mu}+E_{n_\alpha})\right]^{1/2},\\
M_{21,22} &= \left[k_{\perp\alpha}^2+\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\mu}+E_{n_\alpha})\right]^{1/2},\\
M_{23,24} &= \left[k_{\perp\beta}^2+\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\mu}+E_{n_\beta})\right]^{1/2},\\
M_{25,26} &= \left[k_{\perp\alpha}^2+\frac{2m^*}{\hbar^2}(2\hbar\omega\pm\hbar\omega_{LO}-E_{n_\beta}+E_{n_\mu})\right]^{1/2},\\
F_{11} &= (1+N_q)f_\beta\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{11}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad -N_q(1-f_\beta)\big(1+\exp\big[\theta(\frac{\hbar^2M_{11}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
F_{12} &= (1+N_q)(1-f_\beta)\big(1+\exp[\theta(\frac{\hbar^2M_{12}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\\
&\quad -N_qf_\beta\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2M_{12}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big],\end{aligned}$$

$$
\begin{aligned}
F_{13} &= (1+N_q)f_\alpha\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{13}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad -N_q(1-f_\alpha)\big(1+\exp\big[\theta(\frac{\hbar^2M_{13}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
F_{14} &= (1+N_q)(1-f_\alpha)\big(1+\exp[\theta(\frac{\hbar^2M_{14}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\\
&\quad -N_qf_\alpha\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2M_{14}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big],\\
F_{15} &= (1+N_q)(1-f_\alpha)\big(1+\exp[\theta(\frac{\hbar^2M_{15}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\\
&\quad -N_qf_\alpha\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2M_{15}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big],\\
F_{16} &= (1+N_q)f_\alpha\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{16}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad -N_q(1-f_\alpha)\big(1+\exp\big[\theta(\frac{\hbar^2M_{16}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
L_1 &= \int\limits_{-\infty}^{+\infty} I_{n_\mu,n_\gamma}dq_z,\quad L_2=\int\limits_{-\infty}^{+\infty} I_{n_\mu,n_\beta}dq_z,\\
L_3 &= \int\limits_{-\infty}^{+\infty} I_{n_\mu,n_\delta}dq_z,\quad L_4=\int\limits_{-\infty}^{+\infty} I_{n_\alpha,n_\mu}dq_z;\\
F_{21} &= (1+N_q)(1-f_\alpha)\big(1+\exp[\theta(\frac{\hbar^2M_{21}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\\
&\quad -N_qf_\alpha\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2M_{21}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big],\\
F_{22} &= (1+N_q)f_\alpha\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{22}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad +N_q(1-f_\alpha)\big(1+\exp\big[\theta(\frac{\hbar^2M_{22}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
F_{23} &= (1+N_q)(1-f_\beta)\big(1+\exp[\theta(\frac{\hbar^2M_{23}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\\
&\quad -N_qf_\beta\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2M_{23}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big],\\
F_{24} &= (1+N_q)f_\beta\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{24}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad -N_q(1-f_\beta)\big(1+\exp\big[\theta(\frac{\hbar^2M_{24}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
F_{25} &= (1+N_q)f_\beta\big[1-\big(1+\exp[\theta(\frac{\hbar^2M_{25}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}\big]\\
&\quad -N_q(1-f_\beta)\big(1+\exp\big[\theta(\frac{\hbar^2M_{25}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1},\\
F_{26} &= (1+N_q)(1-f_\beta)\big(1+\exp[\theta(\frac{\hbar^2M_{26}^2}{2m^*}+E_{n_\mu}-E_F)]\big)^{-1}
\end{aligned}
$$

$$-N_q f_\beta\big[1-\big(1+\exp\big[\theta(\frac{\hbar^2 M_{26}^2}{2m^*}+E_{n_\mu}-E_F)\big]\big)^{-1}\big].$$

Inserting Eq.(7) and Eq.(12) into Eq.(6), we obtain the expression of nonlinear absorption power. It can be seen that the analytical results are significant. However, physical conclusions can be drawn from numerical results and graphical plotting.

## 4 NUMERICAL RESULTS AND DISCUSSIONS

To clarify the obtained results we numerically calculate and graphically plot the nonlinear absorption power $P_{NLn}(\omega)$ for a specific quantum well with triangular potential made up of GaAs/AlAs. The parameters used in our calculations are as follow [13, 23, 24, 25]: $\varepsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m^* = 0.067m_0$ ($m_0$ being the electron rest mass), $\hbar\omega_{LO} = 36.25$ meV, $n_\alpha = 0$, $n_\beta = 1$.

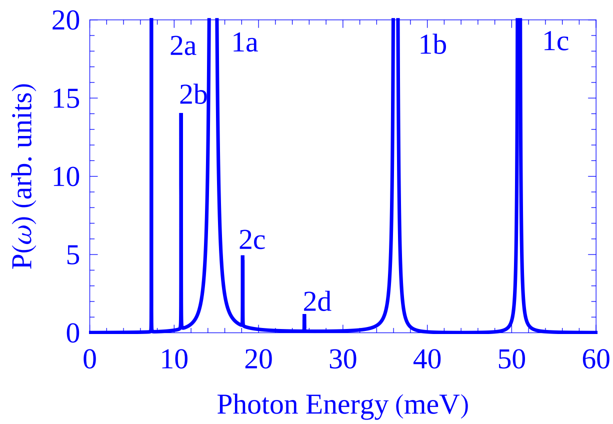


Figure 1: Nonlinear absorption power $P_{NLn}(\omega)$ as a function of photon energy. $T = 200$ K, $E_0 = 10^6$ V/m.

The expression of nonlinear absorption power in Eq.(6) exhibits a resonant behavior due to the ODEPR condition

$$2\hbar\omega \pm E_{\beta\alpha} \pm \hbar\omega_{LO} = 0, \quad \text{or} \quad E_\beta = E_\alpha \pm 2\hbar\omega \pm \hbar\omega_{LO}. \tag{15}$$

Equation (15) is the nonlinear ODEPR condition in quantum well with triangular potential. When the nonlinear ODEPR conditions are satisfied, in the course of scattering events, electrons in the state $|\alpha\rangle$ could make transition to state $|\beta\rangle$ by absorbing two photons of energy $\hbar\omega$, accompanied with the absorption and/or emission of a LO-phonon of energy $\hbar\omega_{LO}$.

Figure 1 describes the dependence of nonlinear absorption power on the photon energy at $T = 200$ K, corresponding to $E_{\beta\alpha} = 14.6$ meV. The graph has seven peaks, each of which describes a specific transition. Peaks 1a, 1b and 1c correspond to the linear case and satisfy the linear ODEPR condition:

$$\hbar\omega \pm E_{\beta\alpha} \pm \hbar\omega_{LO} = 0, \quad \text{or} \quad E_\beta = E_\alpha \pm \hbar\omega \pm \hbar\omega_{LO}. \tag{16}$$

Peaks 2a, 2b, 2c, 2d, satisfying the linear ODEPR conditions, can be explain as follows:

Peak 2a, corresponding to the value $\hbar\omega = 7.3$ meV, describes the transition from state $|\alpha\rangle$ to state $|\beta\rangle$ by absorbing two photons $\hbar\omega$ and without absorbing or emitting of a LO-phonon of energy $\hbar\omega_{LO}$. Peak 2b and 2d correspond to the value $\hbar\omega = 10.825$ meV and $\hbar\omega = 25.425$ meV, which satisfy the nonlinear ODEPR condition $E_\beta = E_\alpha \pm 2\hbar\omega \pm \hbar\omega_{LO}$. Peak 2c corresponds to the value $\hbar\omega = 18.125$ meV, which satisfies the condition $2\hbar\omega = \hbar\omega_{LO}$. Therefore, this peak describes the intraband transition. Figure 2 shows the dependence of nonlinear

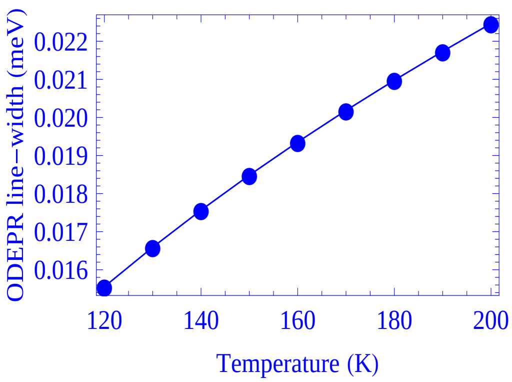


Figure 2: Dependence of nonlinear ODEPR- linewidths on temperature $T$. $E_0 = 10^6$ V/m.

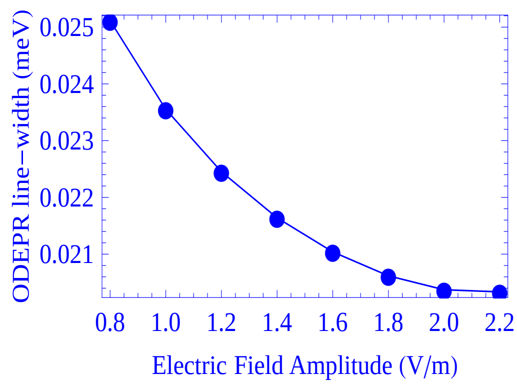


Figure 3: Dependence of nonlinear ODEPR- linewidths on electric field amplitude $E_0$. $T = 200$ K.

ODEPR- linewidths on temperature. From the figure we can find that the linewidths increases with temperature. This result is consistent with the theoretical results of Kang and co-works [2, 3, 4, 5], and of Li and Ning's results [6], and experimental data of Unuma [7]. This can be explained that as temperature increases, the probability of electron-phonon scattering increases, and so do the linewidths.

Figure 3 shows the dependence of nonlinear ODEPR- linewidths on electric field amplitude. It can be seen from the figure that ODEPR- linewidths decrease with electric field amplitude $E_0$. This can be explained that as the electric field amplitude increases the confinement of electron decreases, the probability of electron-phonon scattering decreases, and so do the ODEPR- linewidths.

Figure 4 and 5 are the comparison between the linear ODEPR - linewidths and the nonlinear ones with different values of temperature and electric field amplitude. The nonlinear ODEPR - linewidths are always smaller than linear ones. This can be explained that the probability of absorbing two photons is smaller than the probability of absorbing one photon.

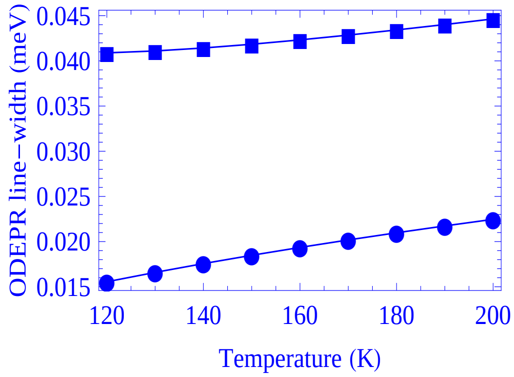


Figure 4: Dependence of linear ODEPR-linewidths (square) and nonlinear ones (round) on temperature $T$. $E_0 = 10^6$ V/m.

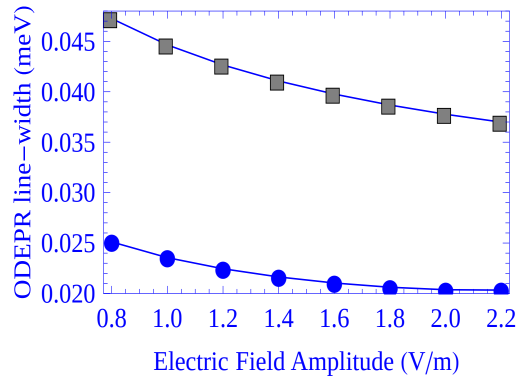


Figure 5: Dependence of linear ODEPR-linewidths (square) and nonlinear ones (round) on electric field amplitude $E_0$. $T =$ 200 K.

## 5 CONCLUSIONS

We have obtained analytic expression of absorption power in quantum well with triangular potential due to electron–LO-phonon interaction. We numerically calculated and plotted graph of $P_{NLn}(\omega)$ for a specific quantum well to clarify the theoretical results and obtained the ODEPR conditions. The importance of the present work is the appearance of resonant peaks satisfying the nonlinear ODEPR conditions. A special attention has been given to the behavior of the ODEPR spectra, such as the splitting of ODEPR peaks from the selection rules. Therefore, they can be applied to optically detect the distance between two energy levels of electrons in quantum well with triangular potential.

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